Research Article

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Comparing the Islamic and Conventional Indices Focusing on Fractal Market Hypothesis

Fraktal Piyasa Hipotezi Odağında İslami ve Konvansiyonel Endekslerin Karşılaştırılması

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Abstract

Being able to accurately predict the direction of price fluctuations in financial instruments and their characteristics within this framework forms the basis for scientifically explaining the functioning of financial markets. The generally accepted theory, which is based on the assumption that security price movements occur on a random walk basis and that returns are normally distributed, is the Efficient Market Hypothesis. The idea that financial instruments may be inadequate in explaining price movements has led to the proposal and testing of the Fractal Market Hypothesis as an alternative in academic circles. In this perspective, the research purpose is to reveal the existence of the Fractal Market Hypothesis in both Islamic and conventional indices. In addition, the fractal properties of the selected indices are evaluated with Trend-Adjusted Fluctuation Analysis (DFA) and Transformed Breadth Analysis (R/S). In the study, daily closing values of six conventional and six Islamic indices generally accepted in the field of international finance between the years 2014 and 2024 are used as dataset. According to the findings, great majority of the Islamic indices exhibit strong long-term dependencies and fractal characteristics. On the contrary, conventional indices exhibit short-term correlations and anti-permanent behaviors. These results illustrate that Islamic markets will provide significant advantages from risk management perspective and long-term investments, especially during financial crisis periods. In addition, this study provides empirical evidence supporting the applicability of FPH to Islamic finance.

Keywords: Islamic Finance, Fractal Market Hypothesis, Islamic Indices, Conventional Indices, Hurst Exponent.
JEL Codes: G10,G14,G15

Özet

Finansal araçların fiyat dalgalanmalarının özelliklerini doğru tespit etmek, finansal piyasaların işleyişini bilimsel olarak açıklayabilmenin temelini oluşturmaktadır. Etkin Piyasa Hipotezi (EPH), finansal araçların fiyat hareketlerinin rassal yürüyüş ilkelerine dayandığını ve getirilerin normal dağıldığı varsayımı üzerine inşa edilmiştir. Finansal araçların fiyat haareketlerini açıklamada yetersiz olabileceği düşüncesi, akademik çeevrelerde alternatif olarak Fraktal Piyasa Hipotezi'nin öne sürülmesine ve test edilmesine yol

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açmıştır. Bu perspektifte, çalışmada hem İslami hem de konvansiyonel endekslerinde Fraktal Piyasa Hipotezi'nin varlığını ortaya koymak amaçlanmaktadır. Ayrıca, Eğilimden Arındırılmış Dalgalanma Analizi (DFA) ve Dönüştürülmüş Genişlik Analizi (R/S) ile seçili endekslerin fraktal özelliklerini de değerlendirilmektedir. Çalışmada uluslararası finans alanında genel kabul görmüş altı konvansiyonel ve altı İslami endeksin 2014 – 2024 yılları arasındaki günlük kapanış değerleri veri seti olarak kullanılmıştır. Elde edilen bulgulara göre, İslami endekslerinin çoğunun güçlü uzun vadeli bağımlılıklar ve fraktal özellikler gösterdiği tespit edilmiştir. Aksine konvansiyonel endeksler ise kısa vadeli korelasyonlar ve anti-kalıcı davranışlar sergilemektedir. Bu sonuçlar, İslami piyasaların özellikle finansal kriz dönemlerinde risk yönetimi ve uzun vadeli yatırımlar perspektifinden önemli avantajlar sağlayacağını göstermektedir. Ayrıca, bu çalışma FPH'nin İslami finansa uygulanabilirliğini destekleyen ampirik kanıtlar sunmaktadır.

Anahtar Kelimeler: İslami Finans, Fraktal Piyasa Hipotezi, İslami Endeksler, Konvansiyonel Endeksler, Hurst Üsteli.
JEL Kodları: G10,G14,G15

Introduction

It is thought that the Islamic Finance perspective, which has developed itself in a fast, dynamic and broad-based manner in global financial circles, will gradually increase its sphere of influence in the coming years. In this context, Islamic markets, which operate alongside traditional financial systems, offer unique solution proposals and alternative financial instruments based on the principles of Islamic law. In this perspective, while the prohibitions on interest (riba), excessive risk-taking (gharar) and gambling (maysir) are followed in Islamic finance, practices that place risk sharing at the focal point of activities are encouraged (Elteir et al., 2013; 87). In accordance with these principles, Islamic securities markets have gained significant momentum in international financial markets in recent years. A particularly notable development within this sector is the rise of Islamic stock indices specifically designed to track the performance of public companies operating in accordance with Sharia principles. These indices undergo a rigorous screening process to ensure that the companies' business activities and financial ratios are compliant with Islamic law. In addition, the dividends distributed are brought into line with religious standards (Zaidi et al, 2015; 241). Global banks and investment companies, including countries with different religions, provide market participants with investment opportunities parallel to Islamic law.

In the last two decades, economics and finance literature has shown great interest in testing the Fractal Market Hypothesis (FMH) of financial markets (Karakaya and Atukalp, 2022; An et al., 2023). FMH stands out as a strong alternative to the traditional Efficient Market Hypothesis (EMH) in shedding light on financial market behavior. This hypothesis is based on the principles of fractal geometry in explaining the nature of financial markets, which are complex, dynamic, and often chaotic. FMH suggests that markets become stable when they encompass a wide range of investors with diverse investment horizons, thus forming a self-organizing system that balances supply and demand (Dar et al., 2017; 154). One of the most important contributions of FPH is its ability to address the limitations of EMH. EMH assumes that markets are in a constant state of equilibrium and that prices reflect all accessible information. However, as numerous market anomalies and financial crises have indicated, this is not always the case (Karp and Vuuren, 2019; 1). The Fractal Market Hypothesis describes how anomalous occurrences occur and how market trends persist despite the diversity of market participants and their varying investment horizons.

The FMH provides an economic and mathematical framework for fractal market analysis. This hypothesis allows us to perceive the reasons for the existence of self-similar statistical structures and how risk is distributed among investors (Peters, 1994; 39). In this context, Table 1 includes the fractal classification of the series.

Table 1. Fractal Classification of Series

Behavior type	Colour	Hurst Exponent (H)
Non-permanent, deviating from the mean, negatively correlated	Pink Noise	0<H<0,5
Normal Distribution (Gaussian Process)	White Noise	H=0,5
Brownian Motion (Wiener Process)	Brown Noise	H=0,5
Continuous, trend strengthening (Hurst Process)	Black Noise	0,5<H<1
Cauchy process (Cauchy distribution)	Cauchy Noise	H=1

Source: Mulligan, 2004; 158

FPH offers a more realistic perception of market dynamics incorporating the concept of long-term memory and persistent trends. This situation is symbolized by the Hurst exponent, which measures the degree of long-term dependence in the time series. When the Hurst exponent is greater than 0.5, it is interpreted as persistent behavior, and when it is less than 0.5, it is interpreted as anti-persistent behavior (Garcin, 2019; 1950024-1). Thus, with the ability to capture long-term memory, more effective modeling and prediction of market behavior is possible.

Fractal Market Hypothesis provides significant contributions in the stages of risk management and in the development of investment strategies by taking into account different frameworks. Traditional models often fall short in predicting market volatility and risks associated with extreme events. FPH, on the other hand, focuses on fractal dimensions and scaling properties, providing tools for better understanding and prediction of market risks. For example, the fractal dimension measures market complexity and the likelihood of chaotic behavior (Blackledge and Lamphiere, 2022; 2).

Using these insights, traders are capable of instigating more robust trading strategies accounting for the multi-fractal nature of markets. In particular, recognizing that markets are inherently similar across timescales, they can adjust their strategies accordingly to manage risk efficiently and optimize returns. For instance, during periods of high market turbulence, strategies based on FPH can assist identify potential turning points and avoid significant losses.

FPH's emphasis on the inherent scaling properties of financial time series significantly improves market predictions and forecasts. Traditional models assume that return distributions are normal, which does not adequately capture the extremes and anomalies observed in real markets. Nonetheless, FPH addresses the Lévy index, which takes into account the heavy tails and extreme changes in price differences (Liu et al., 2022; 4).

Machine learning models incorporating FPH principles hold promise for more accurate market forecasts. These models simulate future price movements using the Lévy index and other fractal-based metrics, providing valuable insights for both short-term and long-term investment decisions (Blackledge and Lamphiere, 2022; 36). Such advanced forecasting techniques are particularly beneficial in volatile markets such as cryptocurrencies, where traditional models often fail.

FPH makes significant contributions to the literature in understanding and expounding market crises and anomalies. While EPH views crises as anomalies or non-existent events, FPH suggests that crises are a natural consequence of the dominance of certain investment horizons. During financial crises, the fractal nature of markets becomes more outstanding, leading to increased volatility and unpredictability (Kristoufek, 2012; 1).

Studies have revealed that the FPH can effectively explain the behavior of financial markets during crises such as the 2008 Global Financial Crisis. Researchers analyzing stock markets across investment horizons have concluded that the FPH provides a more appropriate explanation for market behavior observed during such turbulent periods (Dar et al., 2017; 153). This understanding will help policymakers and investors develop more effective strategies to mitigate the impact of future crises. Given the increasing importance of Islamic finance, there is still a lack of understanding of the fundamental physical properties of returns in Islamic financial markets. Further research on the fractal nature of these markets can provide critical insights into various important properties such as the distribution of signal singularities, geometric properties, and local scaling behaviors, which are significant for a more profound understanding of the unique dynamics of Islamic financial markets.

As mentioned, the main purpose of this study is to demonstrate the validity of the Fractal Market Hypothesis in Islamic and conventional indices. In this context, five international and one national Islamic indices and five international and one national conventional indices were included in the scope of the research. In addition, to examine the dynamics of Islamic stock markets, Detrended Fluctuation Analysis (DFA) and Transformed Breadth Analysis (R/S) methods were taken into consideration. The contributions of the research to the literature as follows: First, studies on the fractal properties of Islamic markets are quite limited in the international literature. In fact, the majority of these studies used Multifractal Detrended Fluctuation Analysis (MFL) methods (Saâdaoui, 2018; Dewandaru et al., 2015; Bouoiyour et al., 2018). Secondly, in the literature written in Turkish, there are inadequate studies on the Fractal Market Hypothesis (Erdoğan, 2017; Moralı and Uyar, 2018; Çevik and Karaca, 2021; Karakaya and Atukalp (2022), and only Güneş (2020) has been found on the fractal nature of Islamic indices. The most important contribution of the study is that it provides evidence on the fractal nature of Islamic markets, increasing the efficiency of portfolios to be created with these indices and forming the foundation for the implementation of systematic risk management.

In the following sections of the research, a summary of the literature written on the Fractal Market Hypothesis and Islamic indices will be discussed. Then, in the data set and method section, the calculation methodology of the DFA and R/S methods applied in the study will be elaborated on. In the analysis and findings section, the outputs of the methods used will be presented. Finally, in the conclusion section, the analysis results obtained in the previous section will be interpreted.

Literature review

In this section, the literature will be summarized under two headings, namely the Fractal Market Hypothesis and Islamic indices.

Literature review on the Fractal Market Hypothesis

The Fractal Market Hypothesis (FPH) has emerged as a strong alternative to the Efficient Market Hypothesis (EMH), suggesting that financial markets exhibit fractal properties such as long-term dependencies, scaling behavior, and multifractality. In recent years, many academic studies have tested the FPH, contributed to a more efficient understanding of market dynamics and questioned traditional finance theories (Kendirli, 2006; Güneş, 2020; Karakaya and Atukalp, 2022; An et al., 2023).

The development of the Fractal Market Hypothesis (FPH) is based on the pioneering work of Benoit Mandelbrot. Mandelbrot's 1963 paper "The Variation of Certain Speculative Prices" presented a serious challenge to the Gaussian distribution models widely used at the time by introducing the idea that financial markets exhibit long-range dependencies and heavy tails. This concept provided the basis for understanding that financial markets have a fractal structure, that is, that certain patterns repeat at different scales (Mandelbrot, 1963). In another paper conducted by Mandelbrot with Van Ness in 1968, he took this idea further by introducing a mathematical model called fractional Brownian motion (fBm) that captures the long-term dependencies observed in financial time series, and provided an significant framework for FPH (Mandelbrot and Ness, 1968). Towards the end of the 1980s, the application of fractal concepts to financial markets gained momentum with the work of Edgar E. Peters. Peters's 1989 article "Fractal Structure in the Capital Markets" was among the seminal studies to explicitly relate Mandelbrot's fractal geometry to the behavior of financial markets (Peters, 1989). Peters brought significant criticism to the Efficient Market Hypothesis (EMH) by arguing that financial markets exhibit fractal properties. In his subsequent books "Fractal Market Analysis" (Peters, 1994) and "Chaos and Order in the Capital Markets" (Peters, 1996), he developed these ideas further, providing empirical evidence and practical applications of how fractal analysis can be implemented in investment strategies. Peters' work played an remarkable role in the popularization of the Fractal Market Hypothesis and in closing the gap between theory and practice.

The theoretical basis of FMH was significantly influenced by Hurst's (1951) work on long-term dependencies in natural systems. Hurst's findings, especially the Hurst exponent, became the basic tools for measuring the fractal dimensions of financial time series, thus further reinforcing the fractal structure of markets. The empirical validation process of FMH continued with Lux's 1996 study on the stable Paretian hypothesis in German stock returns (Lux, 1996). The author demonstrated the limitations of traditional financial models by showing the existence of heavy tails and long-term dependencies in financial data and contributed to FMH. Interdisciplinary approaches have made significant contributions to the application of fractal concepts to financial markets. Mantegna and Stanley (1995) applied statistical physics to financial indices and demonstrated scaling behaviors, providing additional empirical support for FMH. Bouchaud and Potters (2003) developed these ideas further, integrating statistical physics models into risk management and derivative pricing, thus linking FMH to practical financial applications. Mandelbrot and Hudson (2010) synthesized these developments and provided a comprehensive critique of traditional financial models, reaffirming the importance of fractal structures in understanding market behavior.

Also in the 2000s, Muzy et al. (2000) introduced the multifractal cascade model to explain fluctuations in financial time series, relating multifractality to stochastic volatility and further strengthening the theoretical foundations of FMH. Calvet and Fisher (2008) comprehensively analyzed multifractality in asset returns, demonstrating that financial markets are characterized by multifractal properties where different scaling laws apply in different time periods. This study emphasized the importance of considering the complexity of market dynamics, which is often oversimplified by traditional financial theories.

FPH has also found wide application in the explanation of market crashes and extreme events by Sornette (2004). Using the concepts of critical phenomena and complex systems, Sornette showed that markets exhibit critical points and exponential laws, and that these features are consistent with fractal market behavior. Laherrère and Sornette (1998) further reinforced this view by examining exponential distributions that stretch in economic data and providing empirical evidence for the existence of non-Gaussian, fractal behavior in financial markets.

In the early 2000s, physical scaling laws found wider application in economics. Stanley et al. (2008) studies on scaling and universality in economics provided a theoretical framework that strengthens FPH by revealing the universal nature of fractal structures in economic data. These contributions emphasize the transformation of FPH from a theoretical concept to a robust framework for understanding the complex and non-linear dynamics of financial markets and provide a powerful alternative to traditional finance theories.

In his study, Güneş (2020) aimed to assess the fractal structure of volatility in index data with daily data of

Participation 30 and Dow Jones Islamic Markets Turkey Index between 2013-2020. The analyzes made in line with the purpose provide evidence that there is a fractal structure in both indices. Karakaya and Atukalp (2022) aimed to inspect the existence of a fractal structure in the volatility of stock returns of banks traded in the BIST Bank Index. Based on the analyzes considering the daily returns of the banks in question between January 05, 2010 and December 31, 2020, they obtained findings supporting the existence of the Fractal Market Hypothesis. Similarly, in his study, Niveditha (2024) examined the fractal behavior in the Gold, Bitcoin, Swiss Franc and US Dollar markets in the 2015-2022 sample period using the Continuous Wavelet Transform methodology. The analyzes determined the existence of a fractal structure in the markets in question. An et al. (2023) investigated the nonlinear multi-fractal correlation between carbon price fluctuations and China's economic policy uncertainty (CNEPU) in Shenzhen, Beijing, Tianjin and the national carbon market, and concluded that there are long-range correlations with anti-persistence multi-fractal features in three regional carbon markets.

Literature Review on Islamic Indices

The study of Islamic indices has attracted significant interest among both researchers and practitioners over the last few decades. This interest emerged as an attempt to gain perception on the dynamics, performance and impact of these financial instruments within the broad framework of Islamic finance. Islamic indices consisting of companies that comply with Sharia law, offer a different perspective in the field of investment by strictly adhering to the principles that prohibit interest (riba), excessive uncertainty (gharar) and investments in haram sectors such as alcohol and gambling.

Hussein and Omran (2005) analyzed the performance of the Dow Jones Islamic Market Index (DJIM) by comparing it with conventional indices and found that DJIM did not perform significantly better or worse than conventional indices. While the findings can be interpreted as Sharia-compliant investments and conventional investments can compete, this research has made significant contributions to the acceptance of Islamic indices as a reliable investment alternative. Similarly, Hakim and Rashidian (2002) further deepened the findings of previous studies by investigating the risk and return characteristics of DJIM. As a result of the analysis, it was found that the risk measures of Islamic indices were generally lower than those of conventional indices. It is argued that the exclusion of especially high leveraged companies and speculative firms from Islamic indices supports this situation. It has been claimed that the compliance of the risk aversion tendency with Islamic principles acts as a lifesaver for the market during financial instability processes. In addition, Saiti et al. (2014) examined the performance of Islamic and conventional stock indices during and after the 2008 financial crisis and found that Islamic indices exhibited more stability. This result was attributed to the conservative nature of Sharia-compliant investments. In this context, it can be said that Islamic indices should be seen as safer financial instruments during economic uncertainty processes.

The comparative performance of Islamic and conventional indices has also been the focus of research. Ho et al. (2014) reveal that Islamic indices outperform conventional indices during crisis periods. This finding suggests that Islamic indices offer investors a superior investment alternative, especially during periods of financial instability. However, comparing the performance of Islamic indices with conventional indices during non-crisis periods has been inconclusive. In other words, it has been found that Islamic indices do not exhibit any significant superiority or weakness over conventional indices during these periods. The effect of Islamic indices on portfolio diversification has also been a subject of interest for researchers. Dewandaru et al. (2017) investigate whether the inclusion of Islamic and conventional asset classes in the portfolios of both conventional and Islamic investors will expand the efficient margins of these portfolios. The findings of the study illustrate that in terms of internal asset allocation, both Islamic and conventional fund managers can mutually benefit from Islamic and conventional asset classes in a given asset class under different regimes. The study reveals that the advantages offered by Islamic financial assets in terms of portfolio diversification may depict diversity depending on specific market conditions and asset classes. This situation indicates that portfolio strategies should adopt a dynamic and regime-specific approach, especially for Islamic investors. Another critical aspect of Islamic indices is their performance during periods of market turbulence. Abduh (2020) examined the volatility of conventional and Islamic indices in Malaysia and investigated the impact of the global financial crisis on volatility on both markets. The research findings reveal that the Islamic index is less volatile than the conventional index during crisis periods. In addition, it was found that the financial crisis significantly affected the short-term volatility of the conventional index and the long-term volatility of the Islamic index. These results indicate that Islamic financial markets may be more resilient in crisis situations and may offer an alternative path in terms of risk management. Sakınç and Açıkalın (2022) underline that Islamic financial indices can be preferred as a hedging tool during crisis periods due to their low volatility. Furthermore, Kandemir and Uçar (2023) analyzed the return performances of BIST 50 and BIST 30 conventional indices and Participation 50 and Participation 30 Islamic indices in the pre- and post-pandemic periods using Sharpe, Treynor and Jensen Alpha metrics. According to the findings, it was determined that conventional indices performed better in the pre-Covid-19 period, while Islamic indices performed better in the Covid-19 period.

Konak and Türkoğlu (2022) concluded that the market was not efficient in a semi-strong form as a result of their study in which they revealed the effect of the inclusion of the Participation Index in Borsa Istanbul on stock prices on November 12, 2021. As a result of the analysis made by Delice and Tuncay (2024) for the Dow Jones Global Index and Dow Jones Islamic Market World Index with the EGAARCH method, it was revealed that the index considered within the scope of the Islamic index exhibited higher performance.

Dataset and methodology

The study utilized daily data obtained from the world's and Turkey's leading Islamic and conventional indices (MSCI, MSCI Shariah, Nifty 50, Nifty 50 Shariah, FTSE 100, FTSE Shariah, SveP 500, S&P Shariah, Dow Jones, Dow Jones Shariah, BIST 100 and BIST Participation 100). The existence of fractal structure was investigated through Hurst Exponential coefficients. For this purpose, Rescaled Range Analysis (R/S) and Detrended Fluctuation Analysis (DFA) methods were used. In this direction, it was tried to determine whether the stocks have long memory. The analyzes were performed with Python Jupyter Notebook version 7.0.8.

In this context, the indices included in the analysis, index codes and analysis dates are presented in Table 2. Since each index is analyzed separately, analysis dates and therefore observation numbers may differ. Relevant daily data were obtained from the "Yahoo Finance" database.

Table 2. Analyzed Indices

Indices	Type	Index Code	Dates Analyzed
MSCI	Conventional	MSCI	22.07.2014-22.07.2024
DOW JONES	Conventional	^DJI	22.07.2014-22.07.2024
FTSE 100	Conventional	^FTSE	22.07.2014-22.07.2024
NIFTY 50	Conventional	^NSEI	22.07.2014-22.07.2024
S&P 500	Conventional	^SPX	22.07.2014-22.07.2024
BİST 100	Conventional	XU100.IS	22.07.2014-22.07.2024
MSCI WORLD ISLAMIC MARKET	Islamic	ISWD.L	22.07.2014-22.07.2024
DOW JONES WORLD ISLAMIC MARKET	Islamic	^DJIMI	22.07.2014-22.07.2024
FTSE SHARIAH	Islamic	HLAL	22.07.2019-22.07.2024
NIFTY 50 SHARIAH	Islamic	SHARIABEES	22.07.2014-22.07.2024
S&P 500 SHARIAH	Islamic	^SP500SH	22.07.2014-22.07.2024
BİST Katılım 100	Islamic	XK100.IS	12.11.2021-22.07.2024

Detrended Fluctuation Analysis (DFA)

DFA is a statistical application used to assess the presence of long- term correlations in time series. Originally developed in the field of physiology to analyze heartbeat intervals, DFA has been adapted over time to be applied in various disciplines such as finance. In finance, DFA is of great importance, especially because of its ability to identify and quantitatively measure long-term persistence or memory in financial time series, which traditional methods may not be able to detect due to their inadequacy in handling non-stationary data sets (Peng et al., 1994).

In finance, time series data such as stock prices, exchange rates, and other financial indices often exhibit complex and non-stationary behavior. Such behavior can obscure underlying patterns and make it complicated to both assess inherent risk and predict future movements. DFA addresses this challenge by systematically extracting trends of varying degrees (e.g., linear, quadratic) from the data and then analyzing the residuals to assess the degree of fluctuation at various scales (Kantelhrdt et al., 2002). Outputs of DFA is the fluctuation function, which plotted on a log-log scale, provides a scaling exponent that indicates the existence and strength of long-term correlations. DFA has two main purposes in finance. First, it allows researchers and analysts to determine whether a financial time series exhibits long-term dependencies. This is an indication of persistent trends that may influence future behavior (Carbone et al., 2004). For example, a scaling exponent greater than 0.5 indicates a persistent trend, in which past increases (or decreases) are followed by further increases (or decreases). Conversely, an exponent less than 0.5 indicates a persistent behavior, in which an increase in a value is followed by decrease.

Secondly, DFA is used to improve risk management and forecasting processes in financial markets. By understanding the scaling behavior of financial time series, analysts can more effectively model the risks associated with different assets or portfolios. For example, DFA provides an important tool for distinguishing random walk behaviors (where future movements are not dependent on the past) from more predictable, trend-dependent behaviors (Grech and Mazur, 2004). This distinction is extremely important for developing more accurate models for asset pricing, risk assessment, and portfolio management.

DFA procedure consists of five steps (Teng and Shang, 2018; 312). The first one is to detect the profile of 〖 Xj〗 _ in a series of 〖 N〗_length.

$$X(i) = \sum_{\{j=1\}}^{\{i\}} (x_k - \langle x \rangle), \qquad i = 1, 2, \ldots, N \tag{1}$$

Secondly, the profile is divided into non-overlapping equal segments of length s, such that Ns ≡ ⌊N/s⌋. Thirdly, for each segment v, the local trend of the data will be calculated by the least squares method. Then, the trend-free time series Xs(i) will be definzed as:

$$X_{s(i)} = X(i) - q_{v(i)}, \tag{2}$$

q_v(i) denotes the appropriate polynomial in the segment. The variance for each 2Ns segment is formulated below:

$$F^2_{\{DFA\}}(v,s) = \langle X_s^2(i) \rangle = \left(\frac{1}{s}\right) \sum^{\{s\}} X_s^2\,[(v-1)s + i], \qquad v = 1, 2, \ldots, Ns, \tag{3}$$

$$\langle X_s^2(i) \rangle = \left(\frac{1}{s}\right) \sum_{\{i=1\}}^{\{s\}} X_s^2\,[Ns+1, Ns+2, \ldots, 2Ns], \tag{4}$$

X_s(i) is obtained by averaging over all data points **i** in segment V. Fourtly, the fluctuation function is got by averaging over all segments.

$$F_{\{DFA\}}(s) = \sqrt{\left| \left(\frac{1}{2Ns}\right) \sum_{\{v=1\}}^{\{2Ns\}} F^2(v,s) \right|}. \tag{5}$$

Finally, F_DFA (s) will be examined on the log-log plot to detect the scaling behavior of the fluctuation functions. If the series x_k illustrates a long-term power law correlation, F_DFA (s) increases as a power law at large s.

$$F_{\{DFA\}}(s) \sim s^{\alpha}. \tag{6}$$

Rescaled Range Analysis (R/S)

R/S has a common purpose with the DFA mentioned above. R/S analysis and DFA have similarities, especially in the purpose of detecting and quantitatively measuring long-term correlations in time series data. Both methods are based on fractal theory and are widely employed to estimate the Hurst exponent which reflects the degree of memory or permanence of a time series. The goal of both methods is to calculate the Hurst exponent. However, they differ from each other in terms of approach and application, especially in the processing of non-stationary data.

Hurst (1951) devised the R/S analysis technique, which is used to quantify the permanence or anti-permanence of a time series. Using this technique, the time series is divided into non-overlapping segments. The range of cumulative departures from the mean within each segment is then calculated, and this range is normalized based on the segment's standard deviation. The Hurst exponent is then found by plotting the rescaled interval (R/S) against the segment length on a log-log scale and measuring the slope of the resultant line. The capacity of R/S analysis to discover long-term dependency is one of its primary characteristics; this ability is especially helpful for spotting patterns in stationary time series. The way trends are handled in time series is the primary distinction between DFA and R/S analysis. R/S analysis is more susceptible to the effect of exogenous trends and is regarded a more acceptable approach for stationary time series. Conversely, because DFA specifically eliminates these types of trends, it is more suited for non-stationary data (Kantelhardt et al., 2002).

Concerning the sequence of continuously compounded returns {r1,r2,...,rτ}, τ is the length of the forecast period and (r_τ)¯ is the sample mean. The R/S statistic is calculated as follows:

$$(R/S)_\tau \equiv \frac{1}{S_t}\left[\max_{1\le t\le\tau}\sum_{t=1}^{\tau}(r_t-\overline{r_\tau})-\min_{1\le t\le\tau}\sum_{t=1}^{\tau}(r_t-\overline{r_\tau})\right] \quad (2)$$

$$S_t \equiv \left[\frac{1}{t}\sum_t(r_t-\overline{r_\tau})\right]^{1/2}$$
(3)

$$(R/S)_\tau = (\tau/2)^H \quad (4)$$

Analysis and Findings

Methods such as DFA and R/S use different techniques to calculate the Hurst exponent and the results may vary. Both methods evaluate the recursive properties of the time series, but the mathematical and computational methods used differ. Therefore, the values obtained from the Hurst exponent may also differ. Analyses were performed implementing Python Jupyter Notebook. The ability to work more effectively on massive data sets and obtain accelerated results gives Python an advantage over traditional methods. Python's performance is superior to other techniques, especially in data processing and time series analysis. In addition, it offers more flexible solutions thanks to its powerful libraries and wide analysis options.

DFA method was applied via Python in the analyses performed to obtain the Hurst exponent. First, the window size was determined and then the cumulative time series was calculated. The logarithms of the variance for each window size were calculated and plotted on the Log-Log graph. The slope in this graph represents the Hurst exponent.

Similar steps were followed in the R/S analysis. First, the window size was selected and the cumulative time series was calculated. The difference between the highest and lowest values for each window was found and standard deviations were calculated. Then, the logarithms of the R/S ratios and the logarithms of the window sizes were plotted to create a Log-Log graph. The Hurst exponent was determined from the slope in this graph.

As a result of all these steps, the Hurst exponents and P-values obtained from the DFA and R/S analyses are presented in Table 3.

Table 3. Hurst exponents and P-values as a result of the DFA and R/S analyses

	DFA analysis results		R/S analysis results	
INDICES	H Exponent	P-values	H Exponent	p-Values
MSCI	0.42	0.000***	0.36	0.0000***
MSCI SHARIAH	0.41	0.000***	0.51	0.0000***
DOW JONES	0.48	0.000***	0.46	0.0000***
DOW JONES SHARIAH	0.53	0.000***	0.50	0.0000***
BİST 100	0.65	0.000***	0.52	0.0000***
BIST KATILIM	0.64	0.000***	0.51	0.0000***
NIFTY 50	0.56	0.000***	0.53	0.0000***
NIFTY 50 SHARIAH	0.54	0.000***	0.52	0.0000***
S&P 500	0.48	0.000***	0.33	0.0000***
SveP 500 SHARIAH	0.45	0.000***	0.35	0.0000***
FTSE 100	0.48	0.000***	0.32	0.0000***
FTSE SHARIAH	0.52	0.000***	0.51	0.0000***

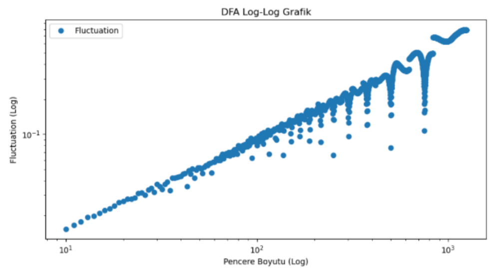
***,**,* indicate 1%, 5% and 10% significance levels respectively.

It was determined that the probability values of the selected conventional and Islamic Indices were statistically significant. H=0.65 was found in the DFA analysis for the BIST 100 index, and H=0.52 in the R/S analysis. Both values indicate trend continuity, but the DFA value of H=0.65 suggests that the future trends of the time series will continue more strongly than the DFA method. This index is in the 0.5<H<1 range and is in the black noise category with a continuous and strong trend. Therefore, it can be said that BIST 100 has a long-term memory. The same situation applies to the BIST Participation Index. The Hurst exponents obtained in both methods are in the black noise range. On the other hand, the H=0.41 value in the DFA analysis for the MSCI Shariah Index reflects the recursive properties of the series and shows that there is negative recursion (anti-persistence) in the series. This suggests that the time series may tend to reverse trends rather than follow a general trend. In the R/S analysis, the value of H=0.51 indicates that the time series may continue its past trends to some extent. Both results indicate that the series has some recursive properties. However, the value of H=0.41 in the CFA suggests that the future trends of the series may be less strong.

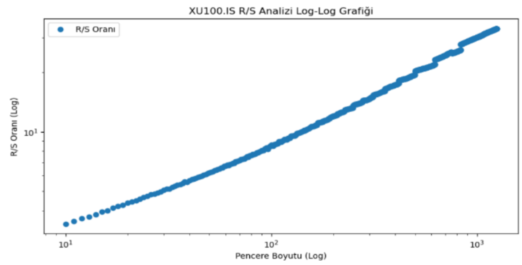
S&P 500 and S&P Shariah Indices, in contrast, show a non-permanent, short-term dependence and negative correlation in both analyses. These indices are in the pink noise category. FTSE 100 and FTSE Shariah Indices, on the other hand, exhibit a different picture. While FTSE 100 Index is in the range of 0<H<0.5 in DFA and R/S analyses and follows a negative recursive, non-permanent trend, FTSE Shariah Index is in the range of 0.5<H<1 in both methods and is in the black noise category that reinforces the continuous trend.

Log-Log graphs related to the findings obtained as a result of the analyses are presented below.

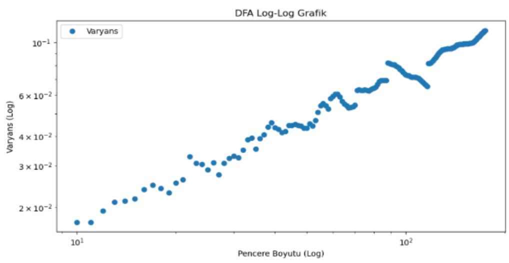
Graph 1. BIST 100 DFA Log-Log graph



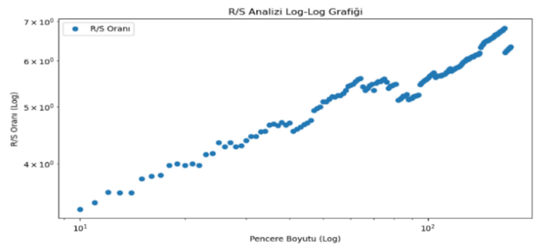
Graph 2. BIST 100 R/S Log-Log graph



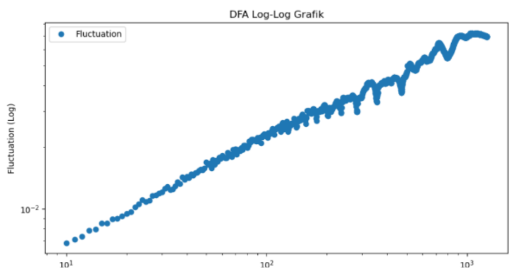
Graph 3. BIST Participation DFA Log-Log graph



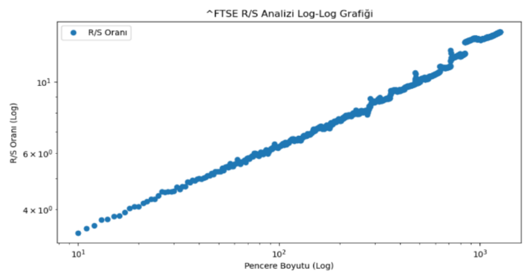
Graph 4. BIST Participation R/S Log-Log graph



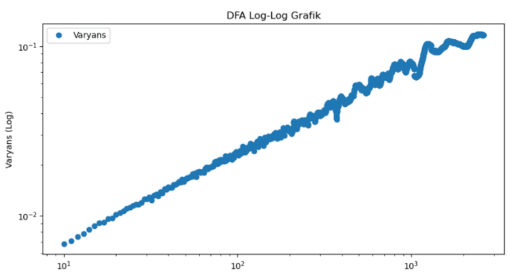
Graph 5 Dow Jones DFA Log-Log graph.



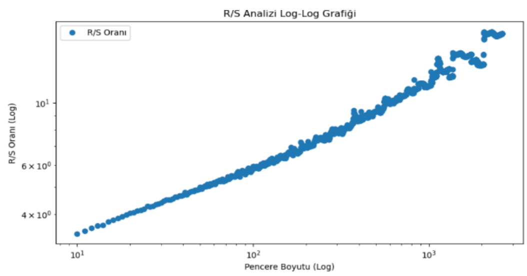
Graph 6. Dow Jones R/S Log-Log graph



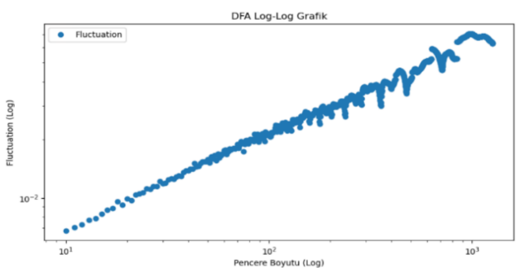
Graph 7. Dow Jones Shariah DFA Log-Log graph



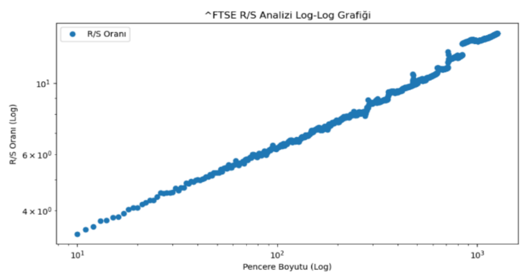
Graph 8. Dow Jones Shariah R/S Log-Log graph



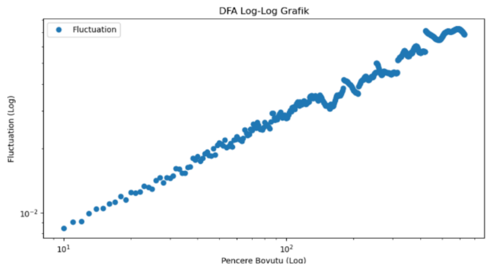
Graph 9. FTSE 100 DFA Log-Log graph



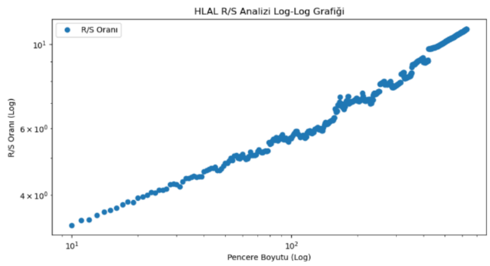
Graph 10. FTSE 100 R/S Log-Log graph



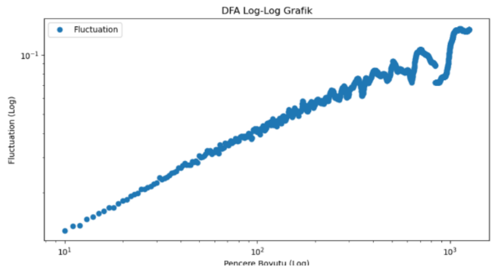
Graph 11. FTSE Shariah DFA Log-Log chart



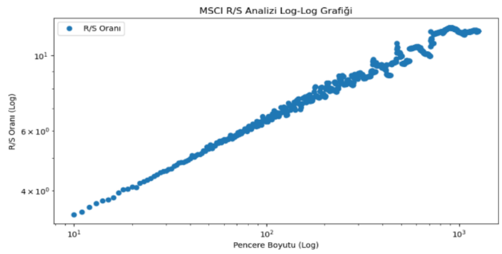
Graph 12. FTSE Shariah R/S Log-Log chart



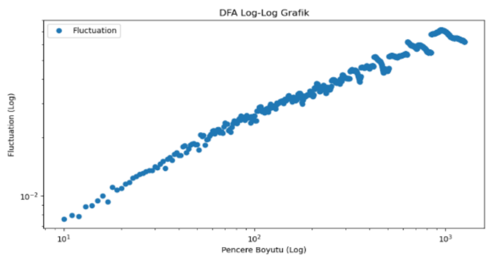
Graph 13. MSCI DFA Log-Log graph



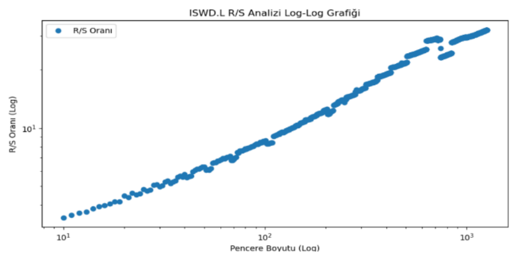
Graph 14. MSCI R/S Log-Log graph



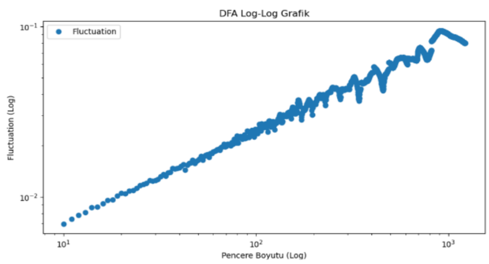
Graph 15. MSCI Shariah DFA Log-Log graph



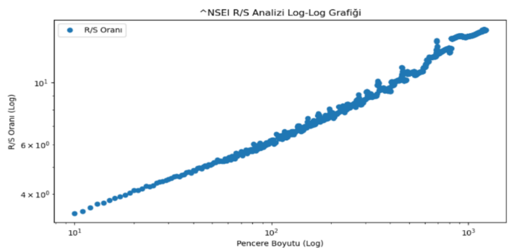
Graph 16. MSCI Shariah R/S Log-Log graph



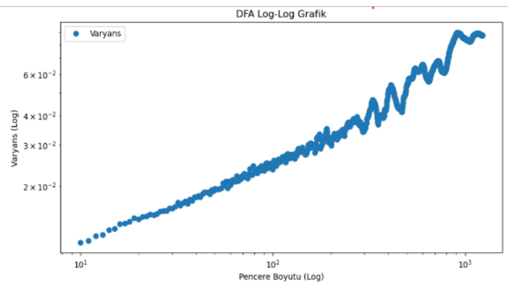
Graph 17. Nifty 50 DFA Log-Log graph



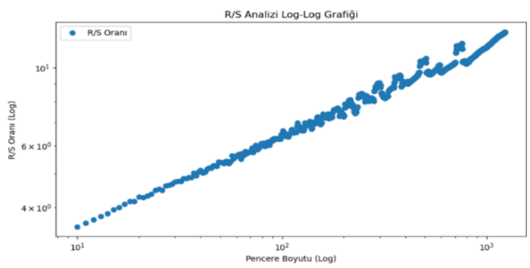
Graph 18. Nifty 50 R/S Log-Log graph



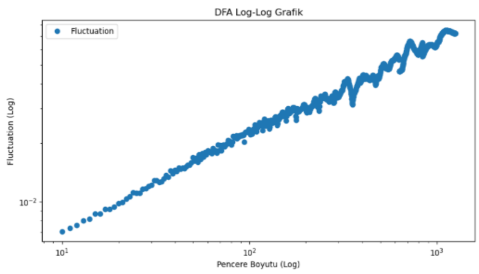
Graph 19. Nifty 50 Shariah DFA Log-Log graph



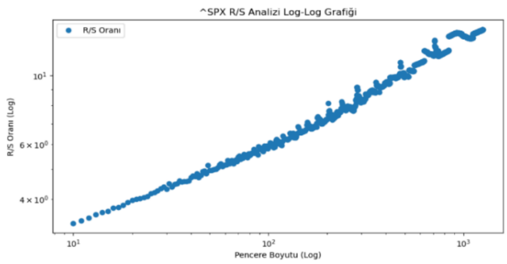
Graph 20. Nifty 50 Shairah R/S Log-Log graph



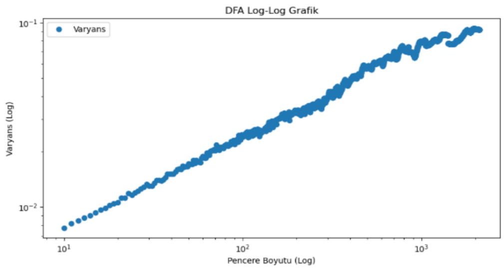
Graph 21. S&P 500 DFA Log-Log graph



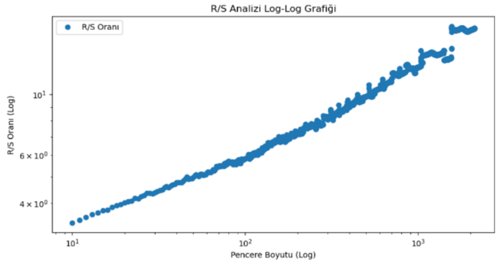
Graph 22. S&P 500 R/S Log-Log graph



Graph 23. S&P Shariah DFA Log-Log graph



Graph 24. S&P Shairah R/S Log-Log graph



Graphs 1-24 contain log-log graphs representing the Hurst exponent obtained for all indices subjected to DFA and R/S analyses. In these graphs, the X-axis (log window size) displays the window sizes on a logarithmic scale. The Y-axis (Log variance) represents the logarithms of the variance values for each window size. The points on these graphs reflect the variances calculated according to various window sizes and are used to analyze the change of these variances. The best line for the data points is found by the regression line on the log-log graph; the slope of this line indicates the Hurst exponent. The value of the Hurst exponent, which is used to assess the series' long-term memory characteristics, is reflected in the slope of the regression line.

Therefore, in the analyses performed assuming that other variables are constant, the reliability of the analysis increases since both methods produce similar results in the Hurst exponent examined using DFA and R/S methods. When the graphs are analyzed carefully, it is seen that 7 out of the 12 indices examined (Graph 16 and 17-MSCI Shariah, Graph 7 and 8-Dow Jones Shariah, Graph 1 and 2-BIST 100, Graph 3 and 4-BIST Participation, Graph 17 and 18-Nifty 50, Graph 19 and 20-Nifty 50 Shariah, Graph 11 and 12-FTSE Shariah) do not exhibit random behavior but have a long-term fractal structure. The other five indices (Graphs 13 and 14-MSCI, Graph 5 and 6-Dow Jones, Graph 21 and 22-S&P 500, Graph 23 and 24-S&P 500 Shariah, Graph 9 and 10-FTSE 100) exhibit pink noise behavior with temporary negative correlation. With the Hurst exponent obtained with DFA and R/S methods, future behaviors of time series are estimated based on their past performances and structural features. It should be emphasized that it is important to evaluate these estimates together with market conditions, external factors and the characteristics of the time series.

Conclusion

The purpose of the research is to evaluate the Fractal Market Hypothesis (FPH) in both conventional and Islamic indexes. If Islamic indices differ from conventional indices in their fractal behaviors, is the key issue of the research. In order to investigate the fractal characteristics of six Islamic and six conventional indices between 2014 and 2024, Detrended Fluctuation Analysis (DFA) and Rescaled Range Analysis (R/S) methodologies were included in the research. The main findings obtained reveal that Islamic indices generally exhibit stronger long-term dependencies and fractal properties, and these properties become more pronounced especially during financial crisis periods. In contrast, conventional indices exhibit shorter-term correlations and anti-permanent behaviors. These findings indicate that Islamic markets can offer more robust investment opportunities, especially during periods of economic volatility, due to their inherent long-term memory and stability. In this direction, the study shows similar results to the studies of Laherrère and Sornette (1998), Güneş (2020), Karakaya and Atukalp (2022). The fractal dynamics of Islamic financial markets and provides empirical evidence supporting the applicability of FPH to Islamic indices. This has important implications for policy makers, financial analysts, and investors as it highlights that Islamic indices can provide a more stable investment environment during periods of financial instability. In addition, this study challenges the traditional EMH and claims the inclusion of fractal analysis in the assessment of market dynamics.

This study is not free from some limitations. The analysis is limited to a specific set of indices and a defined time period, which may not fully reflect all market dynamics. In addition, the study only focused on the fractal properties of the indices and did not consider other potential market effects. Future research should expand the scope of the analysis by including a wider range of indices and other emerging markets. In addition, combining machine learning techniques with fractal analysis can provide deeper insights into market behavior and increase forecast accuracy.

The current research's results underscore the significance of taking into account fractal features in the analysis markets, particularly within the framework of Islamic finance. By demonstrating the durability and long-term stability of Islamic indices, it argues for a more nuanced approach to market assessments. It also suggests a promising methodology that acknowledges complex and non-linear financial system behaviors and holds promise for future financial modeling and risk management strategies.

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